Stress Distribution for Heavy Embedded Structures

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Introduction

The planning and construction of increasing numbers of nuclear power plants in recent years have resulted in the need for settlement analyses of very heavily loaded [>10 ksf (480 kN/m²)] large diameter [up to 250 ft (76 m)] embedded structures. Settlements under such conditions can be an important consideration in the site selection process.

Calculations of stresses in a soil mass have conventionally utilized solutions from the theory of elasticity—Boussinesq (1) for a surface point load and Mindlin (2) for an embedded point load in a homogeneous isotropic medium. Integrations of point load solutions of Boussinesq and Mindlin for a rectangular uniformly loaded area have been made by Newmark (3) and Skopek (5), respectively.

Westergaard (7) considered the problem of a point load at the surface and inside a semi-infinite homogeneous mass which is reinforced internally so that no horizontal displacements can occur. Such a material more closely represents stratified soils, in which strong layers reinforce soft layers to bring about a greater lateral stress distribution than is indicated by the formulas for isotropic elastic solids.

Integration for the Westergaard surface point load solution for a rectangular area at the surface was presented by Taylor (6). However, solutions for embedded areal loads in a Westergaard material have previously not been available. Such a solution follows.

Also shown are comparisons of computed settlements under typical heavy nuclear power plant buildings based on Boussinesq, Mindlin, Westergaard, and Mindlin-Westergaard (embedded Westergaard) stress distributions.

Analysis

For a rectangular coordinate system, x, y, Z, Westergaard (7) used a transformed coordinate system x, y, z, in which $z = kZ$, $k = \{(1 - 2\nu)/[2(1 - \nu)]\}^{1/2}$, and

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ν = Poisson's ratio, to obtain the following equation for vertical stress du to a force, P, in the interior of a layered medium:

$$\sigma_z = \frac{P}{4\pi}\left(\frac{z-a}{R_1^3} + \frac{z+a}{R_2^3}\right) \dots\dots\dots\dots\dots\dots\dots\dots$$

in which σ_z = vertical compressive stress; P = force acting at a point (x = 0, y = 0, z = a); $R_1^2 = x^2 + y^2 + (z - a)^2$; and $R_2^2 = x^2 + y^2 + (z + a)^2$.

The vertical stress at a depth, Z, under the corner of a rectangular area with dimensions mZ and nZ placed at a depth $Z = a/k$, i.e., at $z = a$, and

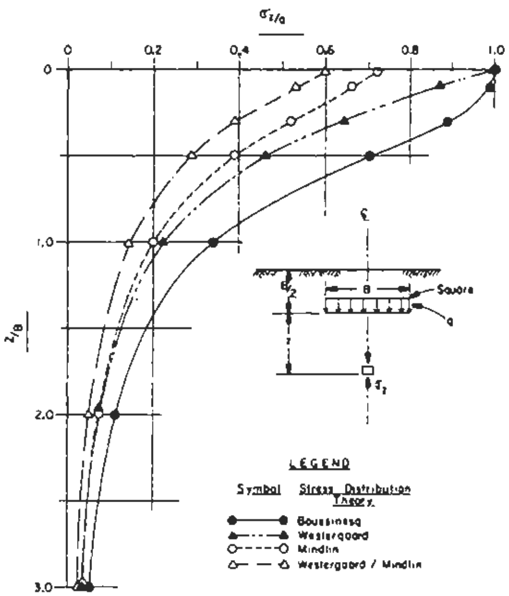

FIG. 1.—Example Stress Distribution below Center of Embedded Square Loaded Area

loaded with a uniform intensity, q, was derived by the writers by integrating Eq. 1, which results in the following expression:

$$\sigma_z = \frac{q}{4\pi}\left[\sin^{-1}\left(\frac{m^2n^2}{m^2n^2 + k_1^2m^2 + k_1^2n^2 + k_1^4}\right)^{1/2} + \sin^{-1}\left(\frac{m^2n^2}{m^2n^2 + k_2^2m^2 + k_2^2n^2 + k_2^4}\right)^{1/2}\right] \dots\dots\dots (2$$

in which $k_1 = k[1 - (a/z)]$; and $k_2 = k[1 + (a/z)]$. Eq. 2 can be programmed for a computer. Stresses in a Westergaard material, under any point below a number of uniformly loaded rectangular areas placed at or below the ground surface, can be obtained by superposition.

COMPARISONS OF CALCULATED STRESSES AND SETTLEMENTS

The stresses due to a uniformly distributed load over a square area with side B, applied at a depth of $0.5B$ below the surface of a semi-infinite medium, were calculated based on the Westergaard-Mindlin (Eq. 2), Boussinesq (6), Westergaard (surface) (6), and Mindlin (4) stress distribution theories. For purposes of comparison, the vertical distribution of stresses under the center

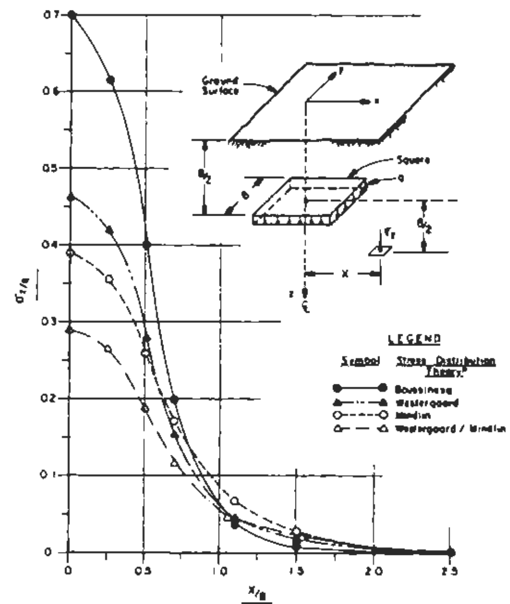

FIG. 2.—Example Stress Distribution along x-Axis below Embedded Square Loaded Area

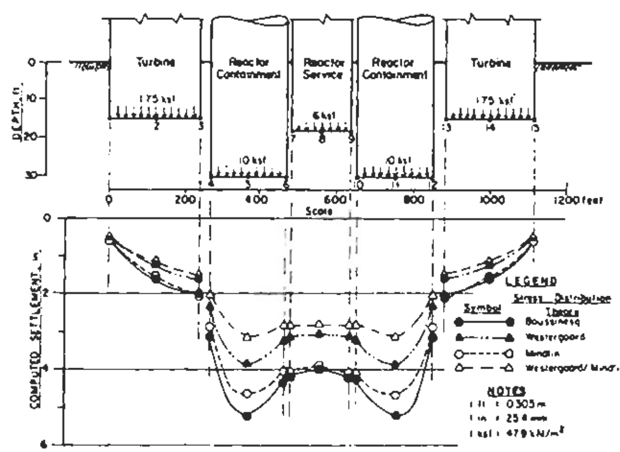

FIG. 3.—Comparison of Computed Settlements

of the loaded area and the horizontal distribution of stresses along a section through the center of the square are shown in Figs. 1 and 2.

To study the effects of various stress-distribution theories on settlements computations were made to estimate the settlements for a group of building placed at various depths below the ground surface for a typical arrangement of a heavy nuclear plant. The buildings, applied loads, and relative elevations as well as computed settlements using the four stress distribution theories, are shown in Fig. 3. For calculation of stresses using Boussinesq and surface Westergaard stress distributions, the ground surface was assumed to be at the base of the structure. The compressibility properties of the soils below the buildings, as well as all other parameters, were kept constant for each computation. Fig. 3 shows that the maximum computed settlement (below point 5) is reduced to 90% of the Boussinesq value when using the Mindlin solution and is reduced to 60% of the Boussinesq value when using the Westergaard-Mindlin solution. Note that for a small depth-to-width ratio, such as for turbines, the differences between the surface and embedded solutions are small, but for deeper embedment, such as for Reactor Containment Buildings, the differences are significant.

Conclusions

Equations are presented for estimating stresses under the corner of a uniformly loaded rectangular area embedded below the ground surface in a Westergaard material. The choice of the stress distribution theory used and consideration of embedment effects for heavily loaded large diameter structures are significant in settlement estimates.

Acknowledgments

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Appendix.—References

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